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On V_{ud} determination from kaon decays

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Abstract: The pion β decay $\pi^+ \to \pi^0 e^+ \nu$ proceeds through pure weak vector hadronic currents and, therefore, the theoretical prediction for it is more reliable than for the processes with axial-vector current contribution. For example, recently the pion β decay has been used for V_{ud} determination. The main aim of this letter is to point that kaon β decay $K^0 \to K^+(\pi^+\pi^0)e^-\bar{\nu}$ analogously can be used for this purpose.

Keywords: Rare Decays, Kaon Physics, Weak Decays.

Contents

1. Introduction

Flavor transitions within and between different quark generations due to the weak charged interactions are described by the Cabibbo-Kobayashi-Maskawa (CKM) unitary mixing-matrix [1, 2]

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1.1}$$

This approach is founded on a pure phenomenological basis and the determination of the matrix elements is completely based on experimental data. If different types of experiments provide a consistent values for a particular matrix element, this should point to the correctness of our results. Therefore, new ideas about systematically independent measurements are welcome.

The subject that we will present concerns the extraction of the well defined matrix element V_{ud} from kaon β decays. The kaon β decay has not been extensively discussed previously in literature. It is even missing from the PDG tables. According to our knowledge for the first time a rough estimation of its decay width has been done in [3]. Other papers [4, 5] suggest to use different final states of this decay to measure the mass difference between neutral and charged kaons, due to the very high sensitivity of the decay to the mass difference value. However, by our opinion, the kaon mass difference is a pure kinematical characteristic and can be measured in other, for example strong interaction processes as the charge-exchange reactions [6, 7]. Therefore, we propose to use the kaon β decay as unique new source of an information about V_{ud} matrix element.

Up to now the most precision determination of this value follows from the superallowed $0^+ \to 0^+$ nuclear β-decay experiments [8, 9]. Based on the data from over 100 different experiments and using a new method for controlling hadronic uncertainties in the radiative correction to superallowed nuclear beta decays along with refinements from [10], Marciano and Sirlin [11] have derived the adopted at present PDG value [12]

$$|V_{ud}| = 0.97377 \pm 0.00027. \tag{1.2}$$

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However, a recent modern determination of the Q-value [9] of the superallowed decay of the radioactive nuclei ^{46}V, obtained from the mass difference of ^{46}V and its decay daughter ^{46}Ti, gives a new Q-value and invalidates the set of its seven previous measurements. This value affects the evaluation of V_{ud} from superallowed nuclear decays and leads to a somewhat lower value for V_{ud}. It may indicate a problem with Q-values of the other superallowed emitters used for V_{ud} determination.

Therefore, independent determination of V_{ud} from other experiments is needed. The second precise evaluation of V_{ud} value, with bigger than superallowed transition uncertainties, is obtained from the measurements of the neutron lifetime and the β asymmetry coefficient A [13]. The later measurements are necessary in order to fix the unknown contribution of the axial-vector nuclear matrix element into the neutron decay rate, which is the main source of uncertainty for V_{ud} extraction.

Using the most precise updated value for the ratio $\lambda^{\rm exp} = g_A/g_V = -1.2733(13)$ [14] of the axial coupling constant to the vector coupling constant and the PDG value for the neutron lifetime $\tau_n = 885.7(8)$ s, one can evaluate V_{ud} as [11]

$$|V_{ud}| = 0.97218 \pm 0.00101. \tag{1.3}$$

This value is 1.5σ lower than the extracted one (1.2) from superallowed nuclear decays and may indicate the presence of new interactions. Their effect, predicted in the [15], leads to the corrected value of $\lambda = -1.2714(13)$ and to more consistent $|V_{ud}| = 0.97339(101)$ value.

However, the extracted from the neutron decays V_{ud} value depends on the experimentally measured neutron lifetime, for which present situation is unclear due to the most recent result [16] $\tau_n = 878.5(8)$. Nevertheless, precision and consistent λ determinations from several correlation coefficient measurements, which are ongoing and planned, would indicate reliable experimental results and would be able to put more stringent constraints on new physics.

And the last but not least important source of information about V_{ud} is the very clean theoretically $0^- \to 0^-$ pion β decay $\pi^+ \to \pi^0 e^+ \nu$. It is a pure vector transition and is free from nuclear structure uncertainties. However, due to the small pion mass difference it has a very weak branch, of the order of 10^{-8}, which leads to severe experimental difficulties. Nevertheless, the PIBETA Collaboration [17], using the Paul Scherrer Institute facilities, has improved the experimental uncertainty for this mode up to 0.6% and quotes

$$|V_{ud}| = 0.9728 \pm 0.0030. \tag{1.4}$$

Therefore, to reach the precision of V_{ud} determination from superallowed nuclear decay (1.2) tenfold improvements both in statistical and systematical errors are necessary. One hopes that with the development of high-intensity proton drivers, this aim can be reached.

It is worth nothing here, that besides excellent possibility for pion and muon physics, these facilities give a unique possibility for kaon physics as well. For example, a CP violation beyond the Standard Model can be searched in rare kaon decays with branching ratios $10^{-10} - 10^{-12}$. One of the background processes is $0^- \to 0^-$ kaon β decay $K^0 \to K^+ e^- \bar{\nu}$, which can give an additional information about V_{ud} value. We are going to discuss this in the next section.

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2. Kaon beta decays

The kaon β decay $K^0 \to K^+ e^- \bar{\nu}$ is completely analogous to the pion beta decay $\pi^- \to \pi^0 e^- \bar{\nu}$. It can serve as a possibility to extract V_{ud} matrix element, because the strange quark s does not participate in the weak interactions and play a spectator role. As far as the final kaon is not a stable particle, it can be registered through its decay modes. From our point of view, the most probable decay channel $K^+ \to \mu^+ \nu$ is not appropriate for the final state identification, because the two neutral neutrinos escape from registration. Therefore, pure hadronic modes, mainly $K^+ \to \pi^+ \pi^0$ decays, are very suitable for this.

It is interesting to note, that the experimental signature of these decays $K^0 \to \pi^+ \pi^0 e^- \bar{\nu}$ does not fulfill $\Delta S = \Delta Q$ selection rule, in contrast to the allowed K_{e4} decays $K^0 \to \pi^- \pi^0 e^+ \nu$, but indicates the presence of $\Delta S = -\Delta Q$ weak transitions. This situation is completely analogous to the experimental puzzle of the beginning of sixties with the observation of $\Sigma^+ \to n \mu^+ \nu$ decays [18], which later have been realized as background [19].

So, let us consider the 'background' events

$$K^0 \to K^+ e^- \bar{\nu} \to \pi^+ \pi^0 e^- \bar{\nu} \tag{2.1}$$

to get a valuable information about the first CKM matrix element. In order to obtain competitive with (1.4) result, we need to keep all uncertainties of the order of 10^{-3}. The amplitude of the first semileptonic decay in (2.1)

$$\mathcal{M} = -\frac{G_F}{\sqrt{2}} V_{ud} \left[f_+(q^2)(p+p')_\mu + f_-(q^2) q_\mu \right] \ell^\mu \tag{2.2}$$

is expressed through the form factors f_+ and f_- of hadronic matrix element $\langle K^+(p') | \bar{u} \gamma_\mu d | K^0(p) \rangle$ multiplied by the leptonic current

$$\ell^\mu = \bar{e} \gamma^\mu (1 - \gamma^5) \nu. \tag{2.3}$$

In general, the form factors depend on the square of the momentum transfer to the lepton pair $q_\mu = (p - p')_\mu$. However, even for the pion β decay the Dalitz plot integral is practically insensitive to this dependence [20], and the form factors can be considered as constants. We can also neglect the form factor f_-, which is proportional to the small isospin mass difference $m_{\pi^+}^2 - m_{\pi^0}^2$, and in SU(2) symmetry limit is equal to zero. Moreover, it is multiplied on the momentum transfer q_μ, which effectively leads to the small contribution in the Dalitz plot distribution, proportional to $(m_e/m_{K^0})^2 \approx 10^{-6}$.

Furthermore, for kaon β decay, in which the initial and final hadrons belong to an $I = 1/2$ multiplet, $f_+ = 1$ with good precision. Isospin corrections in first non-zero approximation are given by the formula [21]

$$\delta f_+ = H_{\pi^+ \pi^0} + 2 H_{K^+ K^-} \approx -6.5 \times 10^{-6}. \tag{2.4}$$

They are negligibly small in accordance to the Ademollo-Gatto theorem [22].

Therefore, the rate of kaon β decay is given by the well-known formula [23]

$$\frac{1}{\tau_{K\beta}} = \frac{G_F^2}{60\pi^3} |V_{ud}|^2 \left(1 - \frac{\Delta}{2 m_{K^0}} \right)^3 \Delta^5 f(\epsilon, \Delta)(1 + \delta), \tag{2.5}$$

where

$$\Delta = m_{K^0} - m_{K^+} = 3.972 \pm 0.027 \text{ MeV} \tag{2.6}$$

is kaon mass difference [12], $\epsilon = (m_e/\Delta)^2$, and the Fermi function f is given by

$$f(\epsilon, \Delta) = \sqrt{1-\epsilon}\left[1 - \frac{9}{2}\epsilon - 4\epsilon^2 + \frac{15}{2}\epsilon^2 \ln\left(\frac{1+\sqrt{1-\epsilon}}{\sqrt{\epsilon}}\right) - \frac{3}{7}\frac{\Delta^2}{(m_{K^0}+m_{K^+})^2}\right], \tag{2.7}$$

while δ represents the effect of the radiative corrections.

The second decay in (2.1) is pure hadronic decay with an experimental branching ratio [12]

$$B(K^+ \to \pi^+\pi^0) = (20.92 \pm 0.12)\%. \tag{2.8}$$

Therefore, the rate of $K^0 \to \pi^+\pi^0 e^-\bar{\nu}$ decay (2.1) can be estimated as

$$\frac{1}{\tau_{K^0\to\pi^+\pi^0 e^-\bar{\nu}}} = \frac{B(K^+ \to \pi^+\pi^0)}{\tau_{K\beta}} \approx 0.02\ \frac{1}{\text{s}} \tag{2.9}$$

However, K^0 and $\bar{K}^0$ states are not invariant under CP symmetry transformation and do not represent physical states. Instead $|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$ and $|K_L\rangle = p\,|K^0\rangle - q|\bar{K}^0\rangle$ combinations are assigned to the physical mesons K_S and K_L, respectively. In the case of CP invariance, $q = p = 1/\sqrt{2}$, K_S represents CP even and K_L CP odd states. Using PDG fit value for mean life of K_L meson [12]

$$\tau_{K_L} = (5.114 \pm 0.021) \times 10^{-8} \text{ s} \tag{2.10}$$

one can estimate the branching ratio of the sum of the two-step decay (2.1) and its CP transformed

$$K_L \to \pi^+\pi^0 e^-\bar{\nu} + \pi^-\pi^0 e^+\nu \tag{2.11}$$

as

$$B(K_L \to \pi^\pm\pi^0 e^\mp\bar{\nu}(\nu)) = \frac{\tau_{K_L}}{\tau_{K^0\to\pi^+\pi^0 e^-\bar{\nu}}} \approx 10^{-9}. \tag{2.12}$$

3. Discussion and conclusions

In this section we would like to discuss the experimental possibility of V_{ud} extraction from decays (2.11). It is obvious that the target of comparison should be the superallowed β decays of nuclei, which provide the best current determination of this value (1.2). However, as is the case for each new project, it is impossible to get the desired precision at the beginning. Nevertheless, we will show that using kaon β decays, an accuracy, not worse than the one from pion β decays, can be reached.

As far as this decays have very small branching ratio, one needs high-intensity beam provided in average 10^7 K_L-decays per second. Probably, the best place for such measurements is the 50 GeV Proton Synchrotron at JHF. In order to derive the rate of the two-step process (2.1) from (2.12) one needs to know with a good precision the lifetime

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of K_L meson or some of its branching ratios. The present accuracy of the lifetime (2.10) $\delta\tau_{K_L}/\tau_{K_L} \approx 4 \times 10^{-3}$ is already good enough and contributes to V_{ud} error at the level of 2×10^{-3}.

The uncertainty in the determination of the rate of kaon β decays, according to (2.9), comes from the experimental accuracy of the branching ratio (2.8) of hadronic mode of the charged kaon decays $\delta B/B \approx 6 \times 10^{-3}$. Its contribution into V_{ud} error, 3×10^{-3}, is also competitive with pion β decay uncertainty. It may be improved in future, because the last direct measurement of this ratio has been done more than thirty years ago [24].

Speaking about the experimental selection of the rare process (2.1) we should note that the main background to it comes from K_{e4} decay with branching ratio $(5.21 \pm 0.11) \times 10^{-5}$ [25]. The difference in $\Delta S = \Delta Q$ selection rule can be used for discrimination of these decays in the case of tagged K^0, $\bar{K}^0$ beams. However, it cannot help in the case of K_L beam, containing both K^0 and $\bar{K}^0$ meson states. Nevertheless, these decays can be well separated kinematically. First of all, the two pions in the final state of (2.1) come with definite invariant mass $M_{\pi\pi} = m_{K^+}$ from the two-particle K^+ decay and apart from K_L decay point. In the same time K_{e4} process is three body decay and its distribution in $M_{\pi\pi}$ variable has a continuous spectrum with the maximum at 340 MeV and follows decreasing to the end. Good knowledge of the form factors [26] of K_{e4} process allows us to subtract background under the peak from process (2.1) around m_{K^+}.

The last and the main source of the uncertainty in V_{ud} determination comes from the $K^0 - K^+$ mass difference (2.6), which enters into the rate of kaon β decay (2.5) in the fifth power. It leads to inappropriate contribution to $\delta V_{ud}/V_{ud} \approx 1.7 \times 10^{-2}$. Experimental situation at present resembles the one in the 1986 for $m_{\pi^+} - m_{\pi^0}$ pion mass difference, when its uncertainty was almost completely determined by the uncertainty in the neutral-pion mass measurements [27].

Therefore, dedicated experiments for a direct measurement of kaon mass difference $m_{K^0} - m_{K^+}$ in the charge-exchange reactions as [6]

$$K^- p \to \bar{K}^0 n \to \pi^+\pi^- n \tag{3.1}$$

or [7]

$$K^+ d \to K^0 p\, p \to \pi^+\pi^- p\, p \tag{3.2}$$

should be envisaged. At present an ideal place for that is DAΦNE facility at Frascati with its high luminosity e^+e^- collider in the energy region of the ϕ resonance. The low energy charged kaons around 127 MeV from ϕ decays are pretty well above the threshoulds 89.5 MeV and 78.5 MeV for (3.1) and (3.2) reactions, correspondingly. The competing project VEPP-2000 at Novosibirsk, which will cover also the ϕ resonance energy region, is expected to start by 2007 year.

Another way to get an information about the mass splitting between charged and neutral kaons, is to provide separate precise measurements of their masses. So, the weighted average of the uncertainties of charged kaon mass measurements is 5 KeV, but due to serious disagreement between the experimental results a huge scale factor ($S = 2.4$) was introduced to the error [12]

$$m_{K^+} = 493.677 \pm 0.013 \text{ MeV}. \tag{3.3}$$

It may be expected that the experiments as DEAR / SIDDHARTA [28] and MIPP [29] will settle the disagreement between the present input data.

Recent measurements of the neutral kaon mass $m_{K^0} = 497.625 \pm 0.001 \pm 0.031$ MeV [30] and $m_{K^0} = 497.583 \pm 0.005 \pm 0.020$ MeV [31] are dominated by systematical errors and do not improve considerably the twenty years old values of $m_{K^0} = 497.742 \pm 0.085$ MeV [32] and $m_{K^0} = 497.661 \pm 0.033$ MeV [33]. Although the weighted average

$$m_{K^0} = 497.614 \pm 0.015 \text{ MeV} \tag{3.4}$$

of all these measurements has an error smaller than PDG, they lead to a worring scale factor ($S = 1.48$), which points to some inconsistency of the measurements.

The main issue in these measurements are not the statistical but the systematic uncertainties. Therefore, the new kaon experiments at CERN and J-PARC facilities should address this issue in future. Without a concrete experimental setup it is impossible to estimate these uncertainties, but even with modest expectations one may hope that they will be at the level of the present statistical errors of few KeV. It will allow V_{ud} extraction from the kaon β decays with a precision not worse than the one from pion β decays with following improvements in future.

We did not discuss the radiative corrections δ and their uncertainties for kaon β decay. Most probably they can be calculated in the same lines as for pion β decay [20]. Their rough theoretical estimations, using older experimental data on superallowed nuclear β-decays, have been done in ref. [5].

In this letter we have proposed the theoretical possibility to extract V_{ud} matrix element from kaon β decay. We have given only primeval experimental insights for the registration of this process and the estimation of the main uncertainties. Of course, in order to provide an experimental realization of this project, a systematical study of such a project is necessary. It will be interesting to analyze the possibility of such measurements within the proposed project [34] searching $K_L \to \pi^0 \nu \bar{\nu}$ decay.

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